Amplitude Squeezing for M Two-Level Atoms Interacting with a Single Mode Coherent Radiation

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Abstract—*The squeezing of electromagnetic field for M two level atoms exiting in the Dicke state $\left| r, m \right\rangle$ and interacting with a single mode coherent radiation has been investigated under the approximation that numbers of photons are small in compared to number of atoms. The time dependent variance has been calculated and the influence of number of photons and some other parameters are noticed. We also find out small radiation squeezing exists under our approximation.*

1. INTRODUCTION

The squeezing of an radiation field has gained a great deal of interest in view of the possibility of Reduction in the noise of an optical signal below the vacuum limit. Earlier squeezing was studied in academic interest but the low-noise property of squeezed states are of interest in connection with practical applications in optical communication and information theory [1-2], quantum teleportation [3-4], gravitational wave detection [5], quantum cryptography [6] etc.

A lot of work has been done on the generation of squeezed states of electromagnetic field and their experimental detection in various processes such as multi-wave mixing [7-10], higher order harmonic generation [11-13], parametric amplification[14], multi-photon absorption [15] , degenerate hyper-Raman scattering [16] etc. The interaction of radiation with matter show large number of interesting effect and different models have been given to explain this [17-19]. The possibility of squeezing in Jaynes-Cumming model (JCM) was studied in several papers [20-23]. The radiation squeezing in two atom JCM with one and multiphoton transition have been studied by number of authors for different initial field input [24-27].A number of nonclassical phenomenons have been described by this model such as vacuum field Rabi oscillation, subpoissonian photon statistics, squeezing of radiation field and collapses and revival phenomenon [28-33]. Earlier we have studied collapses and revivals phenomenon for M two level atoms interacting with a single mode coherent radiation [33].

The aim of present paper is to study radiation squeezing for M two level atoms interacting with a single mode coherent radiation and also investigate the dependence of squeezing on photon numbers and other parameters. We consider the approximation that numbers of photons are large as compared to atoms.

Let us consider the most general operator q_θ defined as

$$q_\theta = \frac{1}{\sqrt{2}}\left[c e^{-i\theta} + c^\dagger e^{i\theta}\right] \tag{1}$$

For this operator, if $\Delta q_\theta \equiv q_\theta - \langle q_\theta \rangle$, the minimum variances (minimum against variation of θ) are seen to be

$$\left\langle (\Delta\, q_\theta)^2 \right\rangle_{\min} = \frac{1}{2} + \left(\left\langle c^\dagger c \right\rangle - |\langle c \rangle|^2 \right) - \left| \left\langle c^2 \right\rangle - \langle c \rangle^2 \right| \tag{2}$$

q_θ is said to be squeezed if $\left\langle (\Delta\, q_\theta)^2 \right\rangle_{\min} < 1/2$

2. TRILINEAR HAMILTONIAN AND THE EQUATION OF MOTIONS

For a single two-level atom or for *M* two level atoms interacting with a single mode coherent radiation, the Hamiltonian [2] is

$$H = H_0 + H_I\,,\; H_0 = H_F + H_A, \tag{3}$$

where

$$H_F = \omega N,\; H_A = \omega R_z,\; H_I = g\;(c R_+ + c^\dagger R_-),\; N = c^\dagger c, \tag{4}$$

Here subscripts *F*, *A*, *I*, refer to field, atom and interaction respectively. $c, c^\dagger$, $N = c^\dagger c$ are the annihilation, creation and number operators for radiation respectively, $R_\pm$ and R_z are the Dicke's atomic operators for the single atom or collective Dicke operators for the atomic assembly [1] , g is coupling constant and ω is the common atomic and radiation frequency.

For M two level atoms, $R_\pm$ and R can be expressed in two–boson representation [2-3] in the form, $R_+ = a^\dagger b$, $R_- = ab^\dagger$ and $R_3 = \frac{1}{2}(a^\dagger a - b^\dagger b)$, with a and b as the annihilation operators for boson modes corresponding to the upper and lower atomic levels.

If we define $S_+ \equiv a^\dagger c$, $S_- \equiv a c^\dagger$, we can write H_I as

$$H_I = g\ (S_+ c + S_- c^\dagger) \quad (5)$$

we can use the Holstein-Primakoff transformation [4-5] for operators $S_\pm$ and write

$$S_+ \equiv a^\dagger c = d^\dagger \sqrt{N_a + N_c - d^\dagger d} = d^\dagger \sqrt{N_0 - d^\dagger d}\ , \quad (6)$$

$$S_- \equiv a c^\dagger = \sqrt{N_a + N_c - d^\dagger d}\ d = \sqrt{N_0 - d^\dagger d}\ d\ , \quad (7)$$

where N_0 is the value of $a^\dagger a + c^\dagger c$, which is constant. For description in terms of operators d, $d^\dagger$ and $c, c^\dagger$, the state $|n_a, n_b, n_c\rangle$ can be written as $|n_d, n_b\rangle$, because $n_d = n_a$ and $n_a + n_c = n_d + n_c$ is constant and specification of n_d and n_b is sufficient to tell n_a n_b and n_c.

We now consider the approximation, $N_0 \gg N_A = n_a + n_b$, i.e. $\langle n_a + n_c\rangle \gg \langle n_a + n_b\rangle$. Obviously this holds very well when the mean number of photons $\bar{n}$ is much greater than the number of atoms. Under this approximation

$$\sqrt{N_0 - d^\dagger d} \approx \sqrt{N_0}\ [1 - (1/2N_0)\ d^\dagger d\ ] \quad (8)$$

and equation (5) reduce to the lowest contributing order in perturbation

$$H_I = G\ [\ (d^\dagger b + b^\dagger d)\ ], \quad G = g\ \sqrt{N_0} \quad (9)$$

This gives the time evaluation operator in interaction picture as $V_0 = e^{-iH_I t}$, in the lowest order. In the lowest order of perturbation, where time evaluation operator is expressed in terms of b , d , $b^\dagger$, $d^\dagger$ only, we can consider states in d and b modes only and write the state at time t as $|\psi_n(t)\rangle$, where n refers to value of n_c at t = 0. We then have $|\psi_n(0)\rangle = |r+m, r-m\rangle$ and

$$|\psi_n(t)\rangle = e^{-iH_I t}|\psi(0)\rangle = e^{-i G t (d^\dagger b + b^\dagger d)}|r+m, r-m\rangle$$

$$= \frac{1}{\sqrt{r+m!r-m!}}(d^\dagger cosGt - ib^\dagger sinGt)^{r+m}(b^\dagger cosGt - id^\dagger sinGt)^{r-m}|vac\rangle$$

(10)

We define $s \equiv p + q$ and write $q = s - p$. Eq. (10) will then be in the form

$$|\psi_n(t)\rangle = \sum_s (-i)^{r+m+s} D_{s,n}|s, 2r-s, r+m+n-s\rangle,$$

(11)

where $D_{s,n}$ is given by

$$D_{s,n} = \sum_p \frac{\sqrt{r+m!r-m!s!2r-s!}}{p!r+m-p!s-p!r-m-s+p!}(-1)^p (sinGt)^{r+m+s-2p}(cosGt)^{r-m-s+2p}$$

(12)

For coherent radiation $|\alpha\rangle$, the field state will be

$$|\psi(t)\rangle = \sum_n e^{-\frac{1}{2}|\alpha|^2}\frac{\alpha^n}{\sqrt{n!}}\sum_s D_{s,n}|s, 2r-s, r+m+n-s\rangle$$

(13)

3. AMPLITUDE SQUEEZING FOR INTERACTION OF M TWO- LEVEL ATOMS WITH SINGLE MODE COHERENT RADIATION

From equation (2) amplitude squeezing is given by

$$\langle(\Delta q_\theta)^2\rangle_{min} - \frac{1}{2} = C_{11} - C_{01}^2 - |C_{02} - C_{01}^2| \quad (14)$$

where C_{01}, C_{02}, C_{11} are given by

$$C_{01} = \sum_n \frac{e^{-|\alpha|^2}|\alpha|^{2n+1}}{n!\sqrt{n+1}}\sum_s \sqrt{r+m+n-s+1}\ D_{s,n} D_{s,n+1} \quad (15)$$

$$C_{02} = \sum_n \frac{e^{-|\alpha|^2} |\alpha|^{2n+2}}{n!\sqrt{(n+1)(n+2)}} \sum_s \sqrt{(r+m+n-s+1)(r+m+n-s+2)}\, D_{s,n} D_{s,n+2} \quad (16)$$

$$C_{11} = \sum_n \frac{e^{-|\alpha|^2} |\alpha|^{2n}}{n!} \sum_s (r+m+n-s)\, D_{s,n}^{\,2} \quad (17)$$

4. CONCLUSIONS AND DISCUSSION OF RESULT

As an illustration of our result we consider the case for fixed M and different values of $\bar{n}$ with atoms prepared in the different atomic states $|r, m\rangle$. We studied the graph showing variation of variance $\langle (\Delta q_\theta)^2 \rangle_{\min}$ with gt and we clearly see that there are oscillations with oscillation frequency at interval $g\, t_0 = \pi / \sqrt{\bar{n} + \frac{1}{2} + m}$ = 0.15 and minimum of variance is 0.37 or max squeezing is 26%, which is also clear from the expression. We also studied the nature of graph for different values of r, m and n and obtained interesting results.

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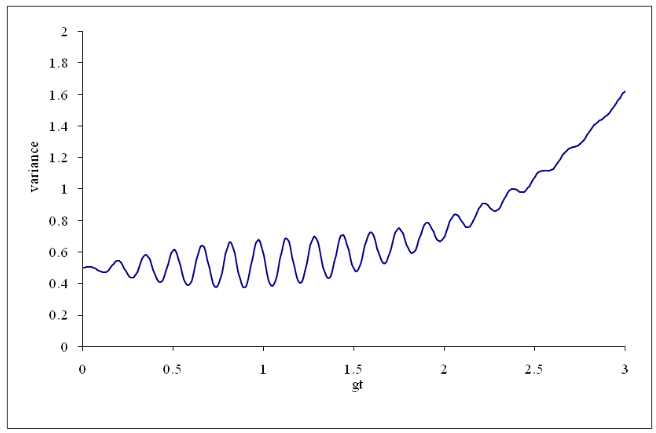

Fig. 1: Graph showing variation of variance $\langle (\Delta q_\theta)^2 \rangle_{\min}$ with $g\,t$ for r = 5 m = 5 $\bar{n}$ = 400.

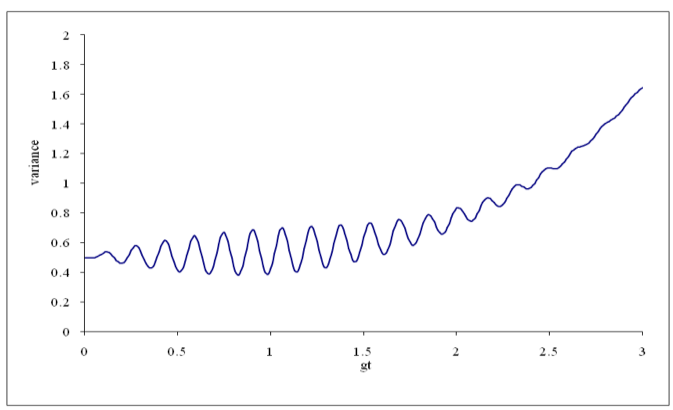

Fig. 2: Graph showing variation of variance $\langle (\Delta q_\theta)^2 \rangle_{\min}$ with $g\,t$ for r = 5 m = -5 $\bar{n}$ =400.